

WAVES ON TITAN

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ABSTRACT

In this paper we take a preliminary look at the possibility of surface waves existing on Titan, assuming it possesses an ethane/methane ocean. The properties of the waves are deduced using well understood theory and compared with the properties of waves on the Earth's oceans. Our calculations show that the lack of knowledge of surface wind speed on Titan produces the greatest uncertainty in the expected wave properties.

Keywords: Waves, Titan, ocean, ethane/methane, Cassini, Huygens.

1. INTRODUCTION

It has been suggested (Ref. 1) that Titan may possess an ethane/methane ocean. The Cassini mission to the Saturnian system will include the Titan atmosphere probe, Huygens, which is expected to survive impact on the surface. The Huygen's payload includes the Surface Science Package (SSP) and Atmosphere Structure Instrument (ASI), which contain accelerometers. Assuming Titan has an ocean, and if waves exist on that ocean, the SSP and/or the ASI should enable wave measurements to be made.

First we need to establish that free surface waves can exist on Titan's ocean. If the densities of the atmosphere and ocean were similar the waves would no longer be free surface waves but internal waves (Ref. 2), such as are found in the interior of the Earth's ocean where there are "layers" of water of different densities. In Table 1 we list some properties of both Titan's and Earth's atmosphere and ocean, and it can be seen that the ratios of atmospheric to oceanic densities (ρ_a / ρ) are fairly similar. In fact, (ρ_a / ρ) are sufficiently similar for waves on Titan to be treated in the same way as terrestrial surface waves.

The purpose of this paper is to take a preliminary look at the properties of surface waves on an ethane/methane ocean. Given the considerably different environmental conditions on Titan, any measurements of waves should allow us to test our wave theories over a different parameter regime (but see below). Moreover, some estimate of the wave properties needs to be made in order that the SSP, ASI and Huygens may be properly designed for the conditions they might encounter.

2. WAVES ON TITAN AND ON EARTH

The theory of waves on the free surface of a fluid has a long history and can be regarded as being well understood. Standard texts (Refs. 2, 3) contain the background necessary for our initial discussion. In contrast, the generation of waves by the wind, a highly nonlinear and random process, is not well understood and we defer discussion of that aspect of the problem to the next section. Here we simply assume that waves exist and deduce some of their properties.

The controlling factors for waves on the free surface of a fluid are gravity, surface tension and viscosity; the relative importance of these factors determining the form of the waves. For waves on the Earth's ocean, gravity is the dominant influence, acting as the restoring force when the surface is disturbed, while surface tension and then viscosity become increasingly important for shorter waves. To determine what the important factors are for waves on Titan we list the properties of the ocean on Titan, as compared with that on Earth, in Table 1.

The viscosity μ of Titan's ocean is much lower than that of Earth's, as is the kinematic viscosity $\nu (= \mu / \rho)$. The decay of waves due to viscous effects (Ref. 3) is dependent on ν , so we can see that viscosity is unimportant for waves on Titan.

It is well accepted (Ref. 2), on the basis of linear theory, that the wave length λ_m dividing the gravity dominated wave regime from the surface tension dominated one is given by

$$\lambda_m = 2 \pi \sqrt{(\tau / \rho g)} \quad (1)$$

where the symbols are as in Table 1. Values of λ_m for Titan and Earth are listed in Table 1, and it can be seen that on Titan, as on Earth, surface tension is only important at very short wavelengths of a few centimetres or less. Thus waves on Titan are dominated by gravity, as are those on Earth.

Assuming that the amplitude to wavelength ratio of the waves is small so that the waves are sinusoidal (linear theory), and that the ocean is deep compared with the wavelength, free surface gravity waves obey the deep water dispersion relationship (Ref. 2) which holds between the wavelength λ and the period T ,

$$\lambda = (g / 2 \pi) T^2 \quad (2)$$

and travel with phase speed $c = \lambda / T$. Thus for a given wave period T , we have

$$\begin{aligned} \lambda_{\text{Titan}} : \lambda_{\text{Earth}} &= g_{\text{Titan}} : g_{\text{Earth}} \quad (3) \\ &= 1.354 : 9.81 \end{aligned}$$

and similarly for c . This means that waves of equal period are shorter and travel slower on Titan than on Earth, reflecting the reduced value of g .

If nonlinear effects are taken into account it is possible to show theoretically (Ref. 3) that the highest steady wave, of a given wavelength, will have 120° corner at the crest (in the absence of surface tension the crest can become a sharp ridge, sloping down either side at an angle of 30° to the horizontal), a crest-to-trough height to wavelength ratio of 0.142 and a phase speed of 1.2 times that of a linear wave of the same wavelength. These, and other results for waves, apply equally well to waves on Titan and on Earth, only taking into account the change in gravity.

Other possible differences in waves on Titan and on Earth could be due to: the rotation rate of Titan, which is slower than that of Earth and can be ignored. A higher atmosphere to ocean density ratio (see Table 1), but this, whilst higher than on Earth, is also small and has been ignored in the above discussion (typically, for surface wave theory the atmospheric density is taken as zero). The depth of the ocean could affect the waves, but if it is greater than one quarter the wavelength its influence is negligible (due to the exponential decay with depth of the wave motion (Ref. 2)). Thus we can conclude that waves on both Titan and on Earth are dominated by gravity, the differences arising primarily from the different values of g .

3. WAVE GENERATION

It is well known, and easily observed, that waves are generated by a wind blowing over the surface of a fluid. However, our understanding of the process of wave generation is still poor, being based on a mixture of theoretical and observational insights. Waves on the Earth's ocean are described statistically, usually in terms of a directional wave frequency spectrum (Refs. 4, 5). The spectrum, which varies spatially and temporally, essentially represents the average amount of energy, at a given wave frequency, travelling in a given direction. The development of the spectrum is described by a wave energy balance equation (Ref. 4), which includes terms that account for wind input to the waves and dissipation (due to wave breaking). Both the wind input term and the dissipation term have been determined semi-empirically, so their applicability to waves on Titan might be questioned. The wind input term is known (Ref. 5) to depend on the ratio of the atmospheric to oceanic densities (ρ_a/ρ), which is higher on Titan than on Earth (see Table 1). This implies a higher rate of energy transfer from the wind to the waves on Titan.

Another factor that might affect the generation of waves on Titan is that the ocean, if it exists, may be planetary in extent. On Earth the continents interfere with the propagation of the waves, but on Titan waves might not encounter any such obstacles. The effect of this difference on the planetary wavefield of Titan is unknown.

In order to make some estimate of the type of waves that might exist on Titan we will make use of the concept of a fully developed sea. This means that a steady wind has been blowing for a sufficiently long time, and over a sufficiently long distance, that a spectral balance has been achieved between wind input and dissipation. Under these conditions the spectrum is fully determined by the wind speed u

(conventionally measured at 10m above the surface) and g . In particular, the period T_p of the waves at the spectral peak can be shown to be (Ref. 6)

$$T_p = c_1 u / g \quad (4)$$

and the significant wave height H_s is given by (Ref. 6)

$$H_s = c_2 u^2 / g \quad (5)$$

where c_1 and c_2 are constants. H_s is defined as four times the square root of the variance of the sea surface elevation, and is a measure of the total energy in the spectrum. The constants have been empirically determined for waves on the Earth's ocean, in the Pierson-Moskowitz (fully developed) case (Ref. 6), to be $c_1 = 7.703$ and $c_2 = 0.242$. Given the change in wind input on Titan (discussed above) it is possible that these values do not apply on Titan. However, as there is no way of evaluating them, until we measure waves on Titan, we will make some estimate of the wave conditions on Titan based on these values. These estimates are given in Table 2. Although models for zonal winds on Titan exist (Ref. 7), nothing is known of the range of wind speed which may be expected on the surface, so wave properties have been derived for a range of values. Table 2 and equations 4 and 5 show that, for the same wind speed, waves on Titan will have a longer period and wavelength, and a greater height than those on Earth, reflecting the decrease in g . However, the average wave steepness (H_s/λ_p) and phase speed (c_p) will be the same.

The results in Table 2 for Titan may well be an underestimate of the values of period, wavelength and significant wave height, as the wind input to the waves is greater on Titan than on Earth (see above). However, the only way to resolve this uncertainty is to measure the waves on Titan.

4. MEASURING WAVES ON TITAN

The most common way to measure ocean waves on Earth is to record the vertical acceleration of a floating buoy. By doubly integrating the signal the vertical displacement of the buoy, as a function of time, is obtained. Cassini will deploy the lander Huygens on Titan. Once it has reached the surface, assuming an ocean exists and it survives the impact and floats, it will act as a buoy and the SSP/ASI accelerometers will allow wave measurements to be made.

On Earth, for routine wave observations it has been common practice to measure the surface elevation every 0.5s for about 20 minutes (1024s) every 3 hours (more recently, 2048s every 1.5 hours). Although survival of the probe is not guaranteed, the forces expected from impact on an ocean at $\sim 6 \text{ ms}^{-1}$ are not large. The nominal probe mission only allows for a few minutes of operation on the surface (the time for which it will be in contact with Cassini once it reaches the surface of Titan). From Table 2 we can see that, for a wind speed of 6 ms^{-1} , in 20 minutes we sample about 200 waves of peak period on Earth, but only about 30 waves on Titan. However, at a sampling frequency of 2Hz our resolution of the waves we sample on Titan is equivalent to sampling on Earth with frequency of 14.5Hz. So, although we do not have many waves of the peak spectral period in our sample, we will resolve their profiles very well. Additionally, we will be able to make good measurements of shorter (higher frequency) waves.

The analysis of the data obtained would include the calculation of the wave frequency spectrum (that is, the spectrum of the wave elevation) and a study of its high frequency tail (which will extend much further relative to the peak, in contrast to the case on Earth, due to the higher

effective sampling rate). Integrals of the spectrum, such as H_s , will be less well determined on Titan than on Earth as a result of the sampling problem discussed above. Another aspect of the analysis would be the comparison of the measured wave profiles with their equivalents on Earth.

In order that the accelerometer be correctly designed it is necessary to make some estimate of the wave accelerations that it will experience on Titan. Fluid particles in a wave move in circles (Ref. 2) (linear theory) and experience an accelerations of magnitude $a\omega^2$, where $\omega = 2\pi/T$ and a is the wave amplitude. Making use of equation 2 we see that

$$a\omega^2 = (2\pi a/\lambda)g \quad (6)$$

If we take $a = H_s/2$, and use the values from Table 2 we obtain an average value of 0.08g, which given the different values of g on Earth and Titan leads to much smaller absolute accelerations on Titan. However, in breaking waves, accelerations of the order of g are possible (Ref. 8), so the SSP/ASI accelerometers needs to be designed to cope with these.

5. CONCLUSIONS

In the above we have explored some of the properties of surface waves on Titan, assuming it possesses an ethane/methane ocean. Measurements of such waves would enable us to test wave theories under very different conditions from those existing on Earth. Surprisingly, perhaps, the above analysis predicts that the type of waves

existing on Titan should not prove to be too different from those on Earth, despite the completely different chemical composition of the atmosphere and ocean. This is because gravity is the dominant influence on the waves in both cases.

6. REFERENCES

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Table 1: Comparison of parameters for Titan and Earth

Parameter	Earth Ocean (@ 20°C)	Titan Ocean	Comment
acceleration due to gravity g	9.81 m s ⁻²	1.354 ± 0.02 m s ⁻²	Based on radius and mass measurements
ocean surface tension τ	7.28 x 10 ⁻³ N m ⁻¹	0.018 ± 0.002 N m ⁻¹	Based on figures for pure methane and pure ethane, assuming a temperature uncertainty of ± 10K
ocean viscosity μ	1.002 x 10 ⁻³ N s m ⁻²	(18±4) x 10 ⁻⁵ N s m ⁻²	As above
ocean density ρ	10 ³ kg m ⁻³	450 - 645 kg m ⁻³	Derived from values for pure methane to pure ethane
atmospheric density ρ_a	1.2 kg m ⁻³	5.3 ± 0.7kg m ⁻³	From model (Ref. 9)
kinematic viscosity ν (μ/ρ)	1.004 x 10 ⁻⁶ m ² s ⁻¹	2.17 - 4.89 x 10 ⁻⁷ m ² s ⁻¹	
ρ_a/ρ	0.0012	0.0071-0.0133	
λ_m	0.0171m	0.0269 - 0.0360m	Capillary/gravity wave divide wavelength

Table 2: Wave parameters for Titan and Earth

Parameter	Titan (Earth)				
	1	3	6	10	20
wind speed u (ms^{-1})					
wave period T_p (s)	5.7 (0.8)	17.1 (2.4)	34.1 (4.7)	57 (7.9)	114 (15.7)
wave length λ_p (m)	7.0 (1.0)	63 (9.0)	250 (34)	700 (97)	2,800 (385)
wave height H_s (m)	0.18 (0.025)	1.6 (0.22)	6.4 (0.89)	18 (2.5)	72 (9.9)
wave steepness H_s / λ_p	0.026 (0.026)	0.026 (0.026)	0.026 (0.026)	0.026 (0.026)	0.026 (0.026)
phase speed c_p (ms^{-1})	1.23 (1.23)	3.7 (3.7)	7.4 (7.4)	12.3 (12.3)	24.5 (24.5)