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Rotating exchange flows through straits with multiple channels: Preliminary results of laboratory studies

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Abstract. The basin exchange flow via a rotating strait with an island separating part of the strait into two channels is investigated. Significant cross-channel or even reverse flow can be expected and no solution may easily be found using existing theoretical approaches usually based on semi-geostrophic theory. Our laboratory experiments have so far confirmed previous findings that rotation reduces the exchange flux and increases across channel slope of the two-layer interface in constant depth channels. Furthermore, significant differences in those parameters to equivalent width channels with an island have been found. Other qualitative differences, such as an influence of the island on the evolution of the initial gravity current, have also been observed. Qualitative results and plans for future comprehensive quantification will be presented.

Introduction

Non-uniform bottom topography across a strait or channel connecting two large basins becomes important when rotation is introduced, or in other words, the Strait is wide relative to the internal Rossby deformation radius. Examples include the Sicily Channel, the Strait of Hormuz and the Cretan Arc Straits, where multiple deep channels are separated by ridges.

Flows around islands or complex topography have been studied in the context of homogeneous or continuously stratified, often atmospheric, fluids [e.g. *Boyer and Davies*, 2000; *Alessio et al.*, 1992] but not for exchange fluxes. Some work on single layer, rotating, zero potential vorticity flow through uniformly sloping channel cross-sections by *Shen* [1981] showed that semi-geostrophic theory for all cross-channel slopes agreed well with experimental results for non-separated flows; however, a stagnation region predicted in the separated case [c.f. *Borenas and Whitehead*, 1998] was observed to break down. Theory not bound to the semi-geostrophic assumption, includes the linear Rossby adjustment problem in a rotating finite width, constant depth channel by *Gill* [1976]; however, information about the steady state is limited. [*Killworth and McDonald*, 1993] developed a maximal flux bound for multi-layer reduced gravity flows in rotating channels of arbitrary topography with non-negative potential vorticity and agrees with *Whitehead et al.* [1974]'s single layer flux for a rectangular, zero potential vorticity and constant depth channel. It is applicable to the island problem but not likely to give additional insight. Although some rotating hydraulic theory for arbitrary topography exists [*Killworth*, 1995], no application to environmental flows has been made as an exact consideration of very complex topography would likely have an infinite amount of virtual controls, making a hydraulic model impractical. To my knowledge, no work has been published regarding the effect of an island on the exchange flow hydraulics through a rotating channel. Such flow will have an additional wall to lean on; in particular, regions near the ends of the island may introduce significant changes in the flow pattern not present in a flat bottom channel. Existing semi-geostrophic theory on rotating two-layer exchange flows, e.g. *Dalziel* [1990], thus cannot easily be applied as the assumption of small cross-channel velocities is violated. However, hydraulic control has been found to occur in many situations, even if specific hydraulic theory is not applicable [*Dalziel*, 1988; *Killworth*, 1995].

We investigate the simplified case of a horizontally contracting (narrows), constant depth, rotating Strait with a full-depth island in the centre (figures 1a and 1b). As a suitable non-dimensional parameter for our problem we define a Rossby number, $R_o = \frac{R}{W}$, based on the Rossby Radius of deformation, $R = \frac{\sqrt{g'H}}{f}$, where f is the Coriolis parameter,$g' = \frac{g\Delta\rho}{\overline{\rho}}$ is the reduced gravity, $\Delta\rho$ the initial density difference between both reservoirs, $\overline{\rho}$ the mean density, W the constricted channel width (subtracting island width) and H the total depth.

Experimental Setup and Scaling

A rotating table-mounted tank was divided into two reservoirs connected by a symmetrically contracting, constant depth channel with a movable island aligned along the channel in the centre. A barrier between the reservoirs, filled with

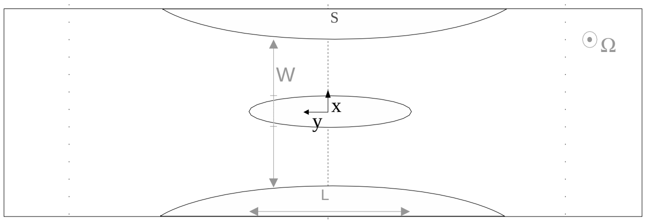

Figure 1a. Plan view of principal geometry and important hydraulic parameters. The coordinate origin is in the centre of the channel with y and x axes as shown. W is the minimum channel width, accounting and Ω=0.5*f is the rotational vector. The dotted line denotes the ends of the channel bordering on the large reservoirs.

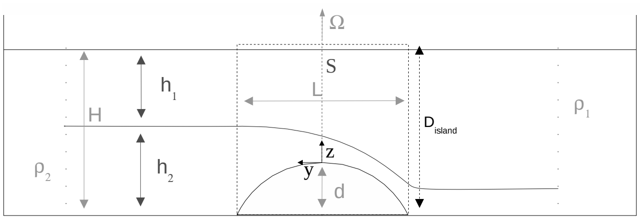

Figure 1b. Side view of figure 1a with coordinate axes as shown. $h_{1(2)}$ and $\rho_{1(2)}$ are the upper (lower) layer depth and density, respectively, and H the total water depth; d is the sill height (d=0 in our experiments to date) and D_{island} the island height; Ω=0.5*f is the rotational vector.

different saline solutions, was removed for a time period sufficient to allow steady lock-exchange at near-initial reservoir conditions. We chose g'˜9.81 cm s^{-2} based on the our apparatus and previous experiments by *Dalziel* [1988]; *Munday* [2000].

Our flows are considered largely inviscid but laminar based on Reynolds number arguments [e.g. p 31 *Tritton*, 1988]. Close to the boundaries, however, viscosity still plays a role giving rise to rotating boundary layers: Ekman layers [e.g. p 228 *Tritton*, 1988] near the floor and interface and Stewartson shear layers near the walls. The former are negligibly small but Stewartson layers cause the interface to flatten near the sidewalls, especially at low rotation. This decreases the channel cross-section available to the exchange flow and increases R_o due to the reduced cross-section available for the exchange: $R_{oeff} = R_o \frac{W}{W-n_S\delta_S}$, where δ_S is the thickness of a single Stewartson Layer, proportional to the Ekman number [p 217 *Tritton*, 1988], and n_S the number of Stewartson layers present in the vicinity of the contraction (n_S=2 for rectangular channel and 4 with an island). Since we have no comprehensive theory at hand to describe the dynamics within the channel and thus the adjoining Stewartson layers the actual Rossby number will be bound by the values of R_o and our R_{oeff}.

Data acquisition

No measurement probes have been used due to known problems with Taylor columns [p 238 *Tritton*, 1988] in homogeneous rotating fluids [e.g. *Boyer and Davies*, 2000]. Instead, fluxes out of each reservoir are calculated from reservoir volumes and initial and final density measurements [see *Whitehead et al.*, 1974]. Density is inferred from temperature measurements and salinity analysis of water samples to an accuracy of 2.3*10^{-5} g cm^{-3}. To study the position of the interface Red food colouring is used as dye in the dense fluid, back-lit from the channel bottom and recorded by a camera and PC based frame grabber. Theory by *Holford and Dalziel* [1996] to make quantitative estimates of interface height is adapted and optimised for our setup. The lower layer thickness, h, can be approximated by assuming an exponential attenuation of the light passing through the fluid from channel bottom; the attenuation coefficient in a uniformly dyed fluid [see p 517 *Apel*, 1987] is $k = \frac{-1}{h} ln(\frac{I}{I_0})$, where I/I_0 is the background corrected absolute light intensity and the path-length of the light through the fluid approximately the same as h (parallax was found to be negligible). A colour filter is used to select that part of the spectrum where the absorptivity function of the dye is linear, i.e. k(c) = Ac, where A is a constant and c the dye concentration. Using the attenuation approximation the layer thickness was directly calibrated allowing h to be accurately measured to within 6% of channel depth.

The two fluids are considered to be immiscible at the interface. Any light scattering effects are included in the dye attenuation and are considered to be negligible. The processing software was DigiImage V1.5 (DL Research Partners, Cambridge) and MATLAB R12.1.

Preliminary Results and Discussion

Treatment of environmental exchange flows often assumes approximately constant reservoir conditions, i.e. well mixed [*Bryden and Stommel*, 1984]. In the lab, finite reservoir volumes lead to a change in reservoir interface heights with time. We have to thus compromise between the timescales underlying our exchange to select data representing the most steady-state flow at nearest to initial reservoir conditions, where hydraulic jumps still isolate the reservoirs from the channel: as soon as the gravity current has passed the end of the channel after approximately $\frac{(g'H)-0.5}{channellength}$, steady flow will begin to set up within approximately f^{-1}. The experiments have thus been taken over an extended time period to find the most ideal observation period and sample spacing using the evolution of cross-channel interface slope in time to detect any passing internal bores.

The exchange flux is defined as, $Q_x = | Q_1 + Q_2 |$ where $Q_{(1,2)}$ are the fluxes out of each of the reservoirs with an associated error of $Q_n = Q_2 - Q_1$, assumed to be zero, so that the inferred flux is $Q_x \pm Q_n$; Q_n was usually similar to the error in Q associated with the density measurements.

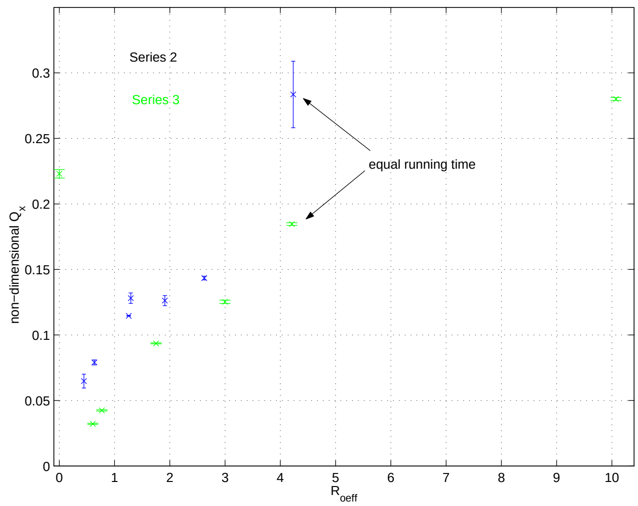

Figure 2. Exchange fluxes (Q_x) vs. effective Rossby number (R_{oeff}). Q_x is non-dimensionalised with respect to internal gravity (Kelvin) wave speed, channel depth and minimum channel width; errors are from net flow Q_n Series 2 has a uniform cross-section and 3 has a centred island. R_{oeff}=0 represents no rotation.

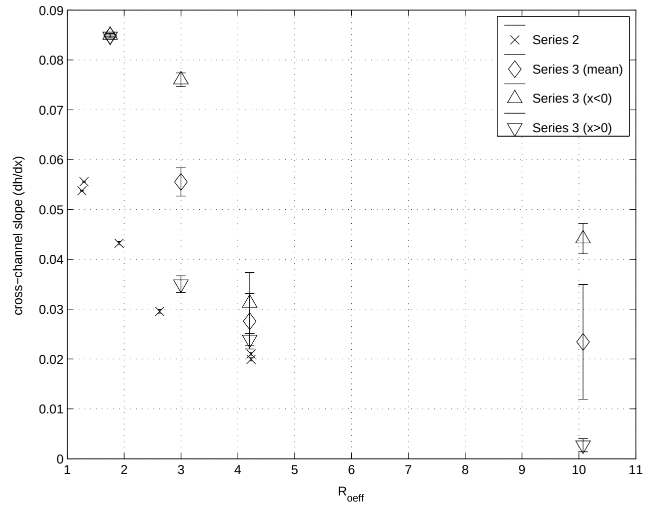

Figure 3. Cross-section slope at the constriction. Series 3 (see figure 2) values for individual channels are also given. Errors from R^2 value of linear fits to slopes.

All Q are non-dimensionalised as described in figure 2. A convenient parameter to compare different fluxes is R_{oeff}, representing the upper bound on the actual Rossby number, as mentioned before. Any difference in Q_x (see figure 2) at the same value of R_{oeff} is likely due to a change in cross-channel geometry, i.e. our island.

The variation of Q_x with R_{oeff} indicates that fluxes generally decrease with increasing rotation. Particularly the island experiments (series 3) show a regularity that appears to be non-linear. Experiments in this series were run for a time period that was scaled by f^{-1} but relatively long compared to the time where initial steady exchange is expected. This means that all experiments can be expected to have reached a time of negligible flux and are thus suitable for an initial comparison.

Running times in non-island experiments (series 2) were not scaled in this way, so that only qualitative observations can be made. However, series 2 shows generally higher fluxes than series 3, which indicates that the presence of an island generally reduces the flux. In particular, two experiments at R_{oeff}˜4, that both had the same running time, so the remaining difference is almost certainly due to the difference in cross-channel floor geometry, i.e. the island.

The cross-channel slope at the constriction section decreases non-linearly with R_{oeff} (figure 3) as is expected from semi-geostrophic theory (see Introduction). In series 3, the slope on the x>0 side of the island varies similarly to series 2 whereas the slope on the other side, although equal in one experiment, appears to differ significantly at R_{oeff}>2. This side is generally associated with a greater slope and a greater error in the linear fit which suggests that it is is more apt to initial disturbances, less likely to be present in the experiment around R_{oeff}˜2 due to a longer running time. This emphasises the importance of using the maximum running time at near-initial conditions. The mean of both slopes on each island side is generally of greater magnitude than the series 2 slopes without the island.

For (R_o,R_{oeff})<1 we observed a curved interface slope that flattened out toward the x<0 side of the channel, giving a more appropriate parameter in the form of the width of a 'Coriolis boundary layer'; this is not to be mistaken for the rotating (Stewartson) shear layers. However, the presumed intersection of the interface with the channel floor did not occur, which was found likely to be due to Ekman layer transport requirements, further explained in *Dalziel* [1988]. Such separated flow is likely to be associated with a channel crossing [*Dalziel*, 1988], occurring within the two-layer region and possibly recirculation or stagnation within the single layer parts of the channel. Crossing of the flow is though to occur within a length R while oscillating along the channel around the constriction at a frequency of ˜f. If R is less than the length of the island, some evidence was found that the crossing occurred in each channel individually with a possible turn around the island tip. Future data also showing these crossing phenomena at high rotation may have to be investigated with respect to frontal dynamics [*Dalziel*, 1988].

We also observed a difference in interface height in the vicinity of the island's sides: the interface on each side did not join but appeared to be lower near the x>0 side than on the opposite one. Even though this agrees with the difference in slope mentioned before, faster rotating experiments did not show this disparity, possibly because they have a different number of Stewartson layers.

Summary and the Future

A laboratory setup and methods for investigating rotating Strait exchange flows with a simple narrows and non-uniform bottom topography has been presented. Methods to attain quantitative flux estimates and interface height measurements have been developed within our setup. Preliminary experiments showed several unique properties for the case of an island centred in the middle of an otherwise flat-bottomed and narrows constricted channel connecting two large fluid exchanging reservoirs. Variation of both interface height and fluxes with respect to the Rossby number suggests the expected influence of rotation but also significant influence by the presence of the island. In particular, maximal fluxes appear to be limited by both increasing rotation and the presence of an island. No direct comparison of fluxes and interface heights to existing theory has been made as the data are not yet sufficient for empirical analysis, but quantitative comparisons with existing work and hydrographic data from the Strait of Sicily are planned for the future.

Future availability of 2-dimensional velocity measurements will allow us to calculate an effective R_o correctly taking the Stewartson layers into account and to investigate variations in potential vorticity. In our setup we expect some finite value, likely constant within each layer and thus related to reservoir depths. Assuming a constant depth channel, the actual value of such potential vorticity is unlikely to affect the flow [*Dalziel*, 1988]. However, this becomes more important if we consider sills as an increase in potential vorticity decreases the critical value of R_o, at which separation occurs. Velocity and h would also allow us to accurately locate hydraulic controls. Some evidence was given by the generally continuous interface slope downward from the dense to the light reservoir. In our setup we would expect a single control to be located at the narrows with supercritical conditions toward the reservoirs, isolated by hydraulic jumps. Only at $R_o\tilde{}1$ two controls may be expected near the channel exits enclosing a sub-critical interface [*Dalziel*, 1988].

Further experimental work will also incorporate an offset of the island toward either channel wall and possibly use of larger turntable facilities to limit the effect of Stewartson layers, deemed insignificant in the ocean.

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References

Alessio, S., L. Briatore, E. Ferrero, A. Longhetto, C. Giraud, and O. Morra, Interaction between atmospheric flows and obstacles: Experiments in a rotating channel, *Boundary-Layer Meteorology*, pp. 235–241, 1992.

Apel, J., *Principles of Ocean Physics*, vol. 38 of *International Geophysics Series*, Academic Press: London, 1987.

Borenas, K., and J. Whitehead, Upstream separation in a rotating channel flow, *Journal of Geophysical Research*, *103*, 7567–7578, 1998.

Boyer, D., and P. Davies, Laboratory studies of orographic effects in rotating and stratified flows, *Annual Review of Fluid Mechanics*, pp. 165–202, 2000.

Bryden, H., and H. Stommel, Limiting processes that determine basic features of the circulation in the mediterranean sea, *Oceanologica Acta*, *7*, 1984.

Dalziel, S., Two-layer hydraulics: maximal exchange flows, Ph.D. thesis, Department of Applied Mathematics and Theoretical Physics, University of Cambridge, 1988.

Dalziel, S., Rotating two-layer sill flows, in *The Physical Oceanography of Sea Straits*, edited by L. Pratt, pp. 343–371, Kluwer Academic, 1990.

Gill, A., Adjustment under gravity in a rotating channel, *Journal of Fluid Mechanics*, *77*, 603–627, 1976.

Holford, J., and S. Dalziel, Measurements of layer depth in a two-layer flow, *Applied Scientific Research*, *56*, 191–207, 1996.

Killworth, P., Hydraulic control and maximal flow in rotating stratified hydraulics, *Deep Sea Research I*, *42*, 859–871, 1995.

Killworth, P., and N. McDonald, Maximal reduced-gravity flux in rotating hydraulics, *Geophysical Astrophys. Fluid Dynamics*, *70*, 31–40, 1993.

Munday, D., Effects of rotation on internal bores, 2000, university of Southampton SOES Honour Research Project Dissertation.

Shen, C., The rotating hydraulics of the open-channel flow between two basins, *Journal of Fluid Mechanics*, *112*, 161–188, 1981.

Tritton, D., *Physical Fluid Dynamics*, 2nd ed., Clarendon Press, 1988.

Whitehead, J., A. Leetmaa, and R. Knox, Rotating hydraulics of strait and sill flows, *Geophysical Fluid Dynamics*, *6*, 101–125, 1974.

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